Numerical Analysis of 3D Eddy Current Fields in Laminated Media under Various Frequencies

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Abstract — A homogenization method for analyzing eddy current fields and resultant loss in laminated media is presented, where the equivalent conductivity tensor proposed in a previous paper is applied. In order to compute fields at various frequencies, the formulas for the effective skin depths along different directions in the anisotropic solid core are derived. The proposed method is numerically investigated in a small analysis model over a very wide frequency range and the calculation results are compared with those from the sheet-by-sheet method using the conductivity and permeability of the iron material and the ordinary homogenization method. It is shown that the proposed method yields basically the same results as the sheet-by-sheet method with much lower computational cost both in linear and nonlinear cases, whereas in some cases the ordinary method results in large errors even at low frequencies.

I. INTRODUCTION

The ordinary homogenization method to analyze fields in laminated cores is always conducted by considering the equivalent conductivity normal to the sheets as zero. The method is only applicable to the low frequency case due to neglecting of the eddy current reaction on magnetic field [1] - [2]. This limitation can be alleviated by combining the 3D non-eddy-current solid model and the 1D eddy-current model of iron sheet [3] - [4]. In this paper, a homogenization method using a practical equivalent conductivity formulation put forward by the authors in previous work [5] is proposed. The 3D eddy currents and the reaction fields are taken into account in the method. The eddy-current reaction field and losses in a simple model are computed using the proposed method over a very wide frequency range and the results are compared with those obtained by the sheet-by-sheet method and the ordinary homogenization method.

II. METHOD FOR FIELD ANALYSIS

A. Anisotropic Equivalent Conductivity [5]

For electrical machines or transformers, the infinitely long laminated iron core model shown in Fig. 1, where the actual 3-D field problem can be reduced to the 2-D problem, is representative. By equating the eddy current loss expressions obtained from the 2-D problem and the 1-D problem in a single iron sheet, the equivalent conductivity tensor is derived as

$$[\sigma]=\begin{bmatrix}\sigma_x & & \\ & \sigma_y & \\ & & \sigma_z\end{bmatrix}=\begin{bmatrix}F\sigma & & \\ & F\sigma & \\ & & (d/a)^2\,\sigma/F\end{bmatrix} \qquad (1)$$

where σ is the conductivity of the iron material, F is the stacking factor, and a and d are the width and thickness of an iron sheet, respectively.

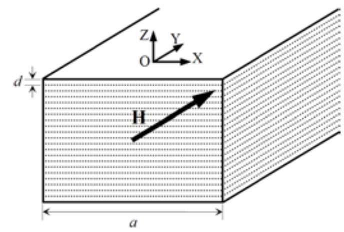

Fig. 1. Laminated iron core with infinite length.

B. Effective Skin Depth

In order to effectively implement meshing of the anisotropic solid iron region at various frequencies, especially at high frequencies, the penetration depths along different directions should be determined in advance. Assuming sinusoidal time variation with the angular frequency ω, the field equation for the 2-D problem indicated in Fig. 1 is

$$\frac{1}{\sigma_z}\frac{\partial^2 H_y}{\partial x^2}+\frac{1}{\sigma_x}\frac{\partial^2 H_y}{\partial z^2}=j\omega\mu_y H_y \qquad (2)$$

where H_y and μ_y are the magnetic field intensity and the equivalent permeability components in the y direction.

It is further supposed that the iron core is also infinite long in the *z* direction and semi-infinite in the *x* direction, and H_y varies merely with *x*. Then (2) is simplified to

$$\frac{1}{\sigma_z}\frac{\partial^2 H_y}{\partial x^2}=j\omega\mu_y H_y\,. \qquad (3)$$

Thus the effective skin depth along the *x* direction is

$$\delta_x=\sqrt{\frac{2}{\omega\mu_y\sigma_z}}\,. \qquad (4)$$

Similarly, the effective skin depth along the *z* direction is

$$\delta_z=\sqrt{\frac{2}{\omega\mu_y\sigma_x}}\,. \qquad (5)$$

The minimum mesh size should be smaller than the skin depth to capture the surface phenomena.

C. Eddy Current Loss

Eddy current loss can be calculated directly in a finite element analysis with the formula

$$P_e=\int_\Omega \frac{J_e^2(x,y,z)}{\sigma_\Omega(x,y,z)}dV \qquad (6)$$

where J_e and σ_Ω are the eddy current density and the equivalent conductivity in field domain Ω .

III. APPLICATIONS AND DISCUSSIONS

A. Analyzed Laminated Iron Core

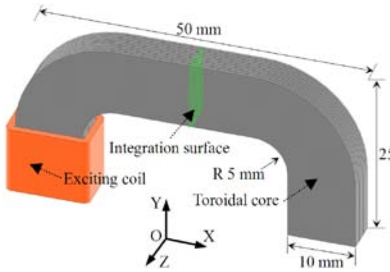

Fig. 2. 1/2 model of laminated toroidal core.

The analyzed toroidal core consisting of 10 electrical steel sheets is shown in Fig. 2, a 100-turn exciting coil being wound around it. Because of the symmetry of the model, only 1/2 of the geometry is analyzed and an integration surface is chosen to calculate the magnetic flux for subsequent analysis. The selected sheet number and core width have been strictly tested to avoid the edge effects and save computing resources. The conductivity of the steel material is 5 MS/m in all cases.

B. Linear Case

The two cases of the 0.5-mm-thick lamination with *F*=0.95 and the 0.1-mm-thick sheet with *F*=0.90 are firstly analyzed. The relative permeability of the steel material is 1000 and the rms value of the sinusoidal exciting current is 0.5 A in the two cases.

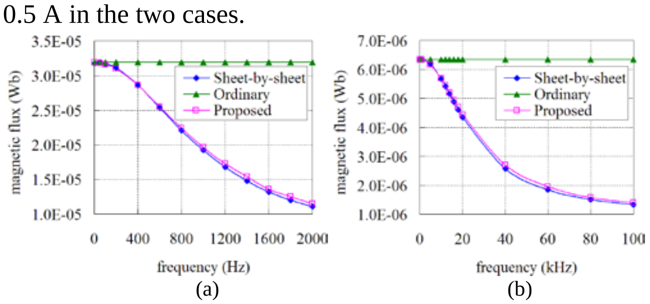

Fig. 3. Magnetic flux amplitude versus depth ratio. (a) 0.5 mm thick iron sheet; (b) 0.1 mm thick iron sheet.

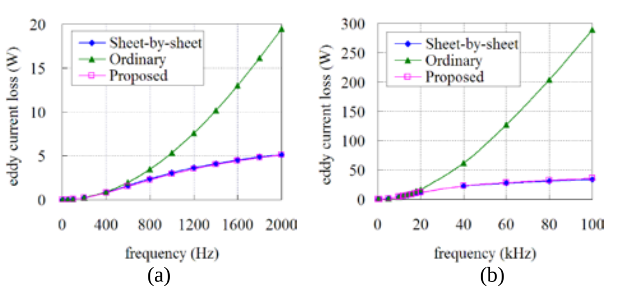

Fig. 4. Eddy current loss versus frequency. (a) 0.5 mm thick iron sheet; (b) 0.1 mm thick iron sheet.

Figs. 3 and 4 illustrate the fact that the magnetic flux amplitudes and eddy current losses computed with the proposed method are in good agreement with those calculated with the sheet-by-sheet method.

On the other hand, when the depth ratio of the iron sheet thickness to the skin depth

$$d \Big/ \sqrt{\frac{1}{\pi f \mu \sigma}} > 1.4 \qquad (7)$$

the fluxes and losses computed with the ordinary homogenization method become significantly greater than the results obtained by the other two methods.

TABLE I
CALCULATED FLUXES AND LOSSES

Method	Frequency (Hz)	Depth ratio	Flux (Wb)	Loss (W)
Sheet-by-sheet			3.82E-5	6.58E-2
Ordinary	100	1.56	4.48E-5	7.55E-2
Proposed			3.83E-5	6.38E-2

Iron sheet thickness: 0.35 mm
Relative permeability: 10000
Stacking factor: 0.95
Exciting current (rms value): 0.1 A

It should be noted that (7) does not occur only under high frequency conditions. As described in Table I, another case for the high permeability steel sheet shows that large errors begin to appear at low frequencies.

C. Nonlinear Case

TABLE II
COMPARISON OF CALCULATED LOSSES AND COMPUTATION TIME

Method	Fundamental frequency (Hz)	Loss (W)	CPU time (mins)
Sheet-by-sheet	50	0.2095	208
Proposed		0.1997	29
Sheet-by-sheet	200	2.1931	431
Proposed		2.1135	41

CPU: Intel Core 2 Dou 3.00GHz
Memory: 2G RAM

In this section, only the sheet-by-sheet method and the proposed method are concerned. The B-H curve and the expression for the non-sinusoidal current will be shown in the full paper. The data reported in Table II show that the proposed method can also obtain nearly the same results as the sheet-by-sheet method with much less computation time.

IV. REFERENCES

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